Implied Volatility Surface by Delta

Document describes IVolatility.com methodology for building Parameterized Implied Volatility Curve (**Parameterized IV skew**) and Implied Volatility Surface by Delta (**Delta surface**).

General

This paper describes IVolatility.com methodology of implied volatility skew data parameterization and Delta surface building. In brief, there are 2 steps in building a Delta surface:

1) parameterize raw implied volatility data for each market expiry (IV strike skew)
2) build Delta surface for a set of standard expiries using this data

Output of the first step is a set of parameters for each market expiry describing a smooth curve of implied variance against log moneyness. We choose it as a parabola and use only out of the money forward (OTMF) options, see a detailed description below. Such method smoothes the "ragged" market data and allows for data compression (IV strike skew is described by just 3 parameters for each expiry).

The second step yields a complete Volatility Surface, that is volatility as a function of period and delta for a fixed set of horizons (30, 60, 90, 120, 150, 180, 360 and 720 calendar days) and deltas (10% to 90% with 5% step), so 9 periods and 17 values of delta total. For the convenience of different applications we also calculate log moneyness and virtual strike for each point - so you are getting IV as a function of delta (or moneyness or even strike) in one dataset - whatever looks more convenient for your application. But note that the dependence of volatility on delta is considered to be "primary", this is explained in detail below.

Normally, each stock has 9x17 = 153 points in its surface for the given trade date. However note that this is not always so: to get a really reliable Delta surface we have to apply quite strict filtering to the input data. For example, for the given expiry we do not build a parabolic curve unless it has at least 5 OTMF options with calculated implied volatility. Half of the US optionable names **do not** pass this filter at all, that is none of the ‘expiries’ in the option chain of those instruments have enough points (basically, these options are not liquid enough). So, roughly 1500 from 3000 US optionable names (as of July 2006) have only at the money points (delta = 0.5) calculated in the surface.
To overcome this difficulty, we also calculate **Raw Delta Surface** dataset. It does not use parameterization results at all, but is built right from the Raw IV data, without any smoothing. The main difference between Raw Delta Surface dataset and our "old" IV Surface by moneyness is that we build it for given standard values of delta, not moneyness. Next section dwells on differences between our implied volatility datasets in more detail.

Difference with other IVolatility.com datasets

The new Delta surface and Raw Delta surface datasets are somewhat similar to other our datasets: IV Surface (by moneyness), IV Index and Raw IV. The table below compares Delta surface, Raw Delta surface and "old" IV Surface (by moneyness).

	Delta surface	**Raw Delta surface**	**IV Surface**
One record contains	IV for a virtual option with given standard expiry and delta; moneyness and strike of this option as an addition	IV for a virtual option with given standard expiry and delta; moneyness and strike of this option as an addition	IV for a virtual option with given standard expiry and moneyness; strike and delta of this option as an addition
Data smoothness	smooth by construction	no smoothing	no smoothing
Coverage	all equity covered by IVolatility.com (all USA plus some European and Canadian names); roughly 50% names have Delta surface data for delta=50% (ATM) only	all equity covered by IVolatility.com (all USA plus some European and Canadian names); roughly 5% names have no IV Surface data	all equity covered by IVolatility.com (all USA plus some European and Canadian names); roughly 5% names have no IV Surface data
Information loss	spikes are smoothed, so most of the "noise" is filtered out; you can restore "smoothed" Raw IV pattern	almost no loss, you can restore Raw IV values almost exactly	almost no loss, you can restore Raw IV values almost exactly
Where calculated	periods 30-720 days, deltas 10-90%	periods 30-720 days, deltas 10-90%	periods 30-720 days, moneyness 50%-150%

As for Raw IV and IV Index dataset, we describe them in a separate table, since they are less similar to three "surface" datasets above:

	IV Index	**Raw IV**
One record contains	IV for a virtual near-ATM option with given standard expiry; composite IV indicator for given stock and expiry	IV for real market option contract; option bid/ask, volume, open interest and greeks as an addition
Data smoothness	manual control of large spikes on a daily basis	no smoothing
Coverage	all equity covered by IVolatility.com (all USA plus some European and Canadian names); roughly 5% names have no IV Index data	all equity covered by IVolatility.com (all USA plus some European and Canadian names); roughly 5% names have no IV for all options
Information loss	far OTM/ITM option data is not taken into account	no loss
Where calculated	periods 30-180	all exchange-listed option contracts

Briefly summing up differences between Delta surface and the other most similar dataset - IV Surface by moneyness:

- Delta surface and Raw Delta surface is built for a standard set of deltas, not moneynesses

- Delta surface (but not Raw Delta surface) is smooth with regard to virtual expiry, strike and historical time by construction
- half of the optionable stocks have "limited" Delta surface - only ATM point for each virtual expiry; Raw Delta surface is calculated and not "limited" for almost all stocks.

The above makes Delta surface far more reliable source of data, however no strike skew can be calculated for about 50% of names here. From the other hand, there is no much meaning in calculating skew for the rest. These are names having poor option chain and illiquid contracts. For them it is typical that implied volatility for even slightly OTM options is not reliable and spiky (changes abruptly from day to day). However, if you are more interested in coverage than in data quality, you can use Raw Delta surface data for these names.

Application of parameterized volatility curves

Parameterized volatility curve has 2 major advantages compared to raw implied data:

- data compression: just 3 parameters describe each expiration instead of point-to-point data
- data regularization: parameterized curve smoothes the original data, which can be too ragged to allow for fine data analysis

Given that, parameterized data is a better choice if one wants to analyze historical skewness of the actual volatility curves. The formula of parameterization (see below) is simple enough to find volatility value for any strike and can be implemented in MS Excel or any other application easily without any programming.

And, of course, this data is used further for building Volatility Surface.

Application of Delta surface

Delta surface is a powerful tool for implied volatility data analysis:

- convenient strike and time skew presentation
- multi-variable presentation: you can look at IV as a function of delta or moneyness or strike, whatever is appropriate for your purpose
- data smoothness: Delta surface is built on the basis of Parameterized IV Skew, which makes it smooth across expiration horizon, delta and historical time - an important factor for precise IV data analysis

And, of course, another important point of using surface data is standardization. To determine if certain option is historically cheap or expensive, one needs to compare it with option in history having same or similar parameters. However, the same expiry and strike can be unavailable in history; the surface provides data for a standard set of periods and deltas for each day in the history. Same point is valid for pair trading and other cross-stock strategies, Delta surface facilitates comparing IV of different stocks.

Why Surface by Delta?

Though we provide moneyness and strike values for each point of the Delta surface, we should emphasize that dependence of IV on delta is "most natural". Roughly, delta is a better indicator (compared to moneyness) of how far out of / in the money the option is. A contract 10% OTM is almost at the money for LEAPs, but a very far OTM for contract expiring in a week. The delta allows tracking this, have a look at the following table:

	spot:	82.35								
DaysTo Expiry	**cStrike**	**moneyness**	**cIVol**	**Delta**	**DaysTo Expiry**	**cStrike**	**moneyness**	**cIVol**	**cDlt%**	**Delta**
9	40	-51%	158	1.00	191	40	-51%	48	100.00	1.00
9	45	-45%	129	1.00	191	45	-45%	40	99.97	1.00
9	50	-39%	108	1.00	**191**	**50**	**-39%**	**36**	**99.78**	**1.00**
9	55	-33%	89	1.00	**191**	**55**	**-33%**	**31**	**99.03**	**0.99**
9	60	-27%	71	1.00	**191**	**60**	**-27%**	**27**	**96.97**	**0.97**
9	65	-21%	54	1.00	**191**	**65**	**-21%**	**24**	**92.68**	**0.93**
9	**70**	**-15%**	**39**	**1.00**	**191**	**70**	**-15%**	**21**	**85.48**	**0.85**
9	**75**	**-9%**	**35**	**0.98**	**191**	**75**	**-9%**	**19**	**75.49**	**0.75**
9	**80**	**-3%**	**27**	**0.75**	**191**	**80**	**-3%**	**17**	**63.55**	**0.64**
9	**85**	**3%**	**23**	**0.27**	**191**	**85**	**3%**	**16**	**50.95**	**0.51**
9	**90**	**9%**	**27**	**0.03**	**191**	**90**	**9%**	**15**	**39.01**	**0.39**
9	95	15%	40	0.00	**191**	**95**	**15%**	**15**	**28.55**	**0.29**
9	100	21%	53	0.00	**191**	**100**	**21%**	**15**	**20.00**	**0.20**
9	105	28%	64	0.00	**191**	**105**	**28%**	**15**	**13.60**	**0.14**
9	110	34%	75	0.00	**191**	**110**	**34%**	**15**	**8.91**	**0.09**
9	115	40%	84	0.00	191	115	40%	18	5.68	0.06
					191	120	46%	20	3.53	0.04

When an option has 9 days to expiration, delta drastically changes from 0.75 to 0.27 between strikes 80 and 85; at the same time moneyness changes only from -3% to 3% here. For expiration further out (191 days) the same strikes' delta changes far less, from 0.64 to 0.51.

The related advantage of choosing delta instead of moneyness is that volatility by delta describes options near the money in more detail.

Finally, Delta surface is "natural" from hedging point of view - you need deltas to hedge, not moneyness.

Methodology and example

This section dwells on a methodology of IV skew parameterization and Delta Surface building. We also provide an example for better understanding here.

Implied volatility parameterization

Parameterization of implied volatility is made by parabolic approximation of raw IV data for each expiry in coordinates x, y:

$$y = a\,x^2 + bx + c\,,$$

Where

$x = Ln(K/F)$ - log moneyness, F - forward price of underlying at expiration;

$y = \sigma^2(K)\,T$ - variance, $\sigma(K)$ - annualized implied volatility for strike K,

T - time to expiration in years.

Only out of the money forward (OTMF) option data is used as parameterization input. The unknown parameterization coefficients a, b, c are found by least squares method i.e. by minimizing expression

$$\sum_K w_K (y_K - a x_K^2 - b x_K - c)^2$$

with respect to a, b, c, where $\{x_K, y_K\}$ - raw coordinates for out of the money forward options with strikes K, w_K - weight function defined as the Black-Scholes probability to find price of underlying at expiration near K:

$$w_K(x_K, y_K) = \frac{\Delta K}{\sqrt{2\pi\, y_K}} \exp\left(-0.5 * \left(\frac{x_K}{\sqrt{y_K}} + 0.5\sqrt{y_K} \right)^2 \right),$$

where ΔK distance between strikes.

Weights w_K are generally greater for near the money options and, parameterization result is less sensitive to far out of the money option data.

In case at the money puts and calls have a gap in volatility, our method removes this gap by shifting put and call IV curve together at point $K = F$ first.

The parabolic parameterization is done only if the number of out of the money options having implied volatility calculated is not less than 5. In other case, we assume that implied volatility pattern is flat, that is $y = c$ and look for this coefficient c ($a=b=0$).

Delta surface building

For a given stock we build parameterization for each available expiration and then calculate IV as the function of delta. We use the standard analytical expression for delta:

$$\Delta(x, y) = \pm \exp(-qT)\, N\left(\mp \frac{x}{\sqrt{y(x)}} \pm 0.5\sqrt{y(x)} \right),$$

Where upper/lower sign corresponds for call/put options, $N(x)$ - the cumulative normal density function,

For fixed Δ and calculated parameterization $y(x) = ax^2 + bx + c$ we find a numerical solution of this equation with respect to x: $x = x(\Delta)$ and thus $y(\Delta) = y(x(\Delta))$.

For each expiry we calculate IV for several fixed delta points: $\Delta = -0.5, \ldots, -0.1$ and $\Delta = 0.1, \ldots, 0.5$ for out of the money puts and calls correspondingly with delta step 0.05.

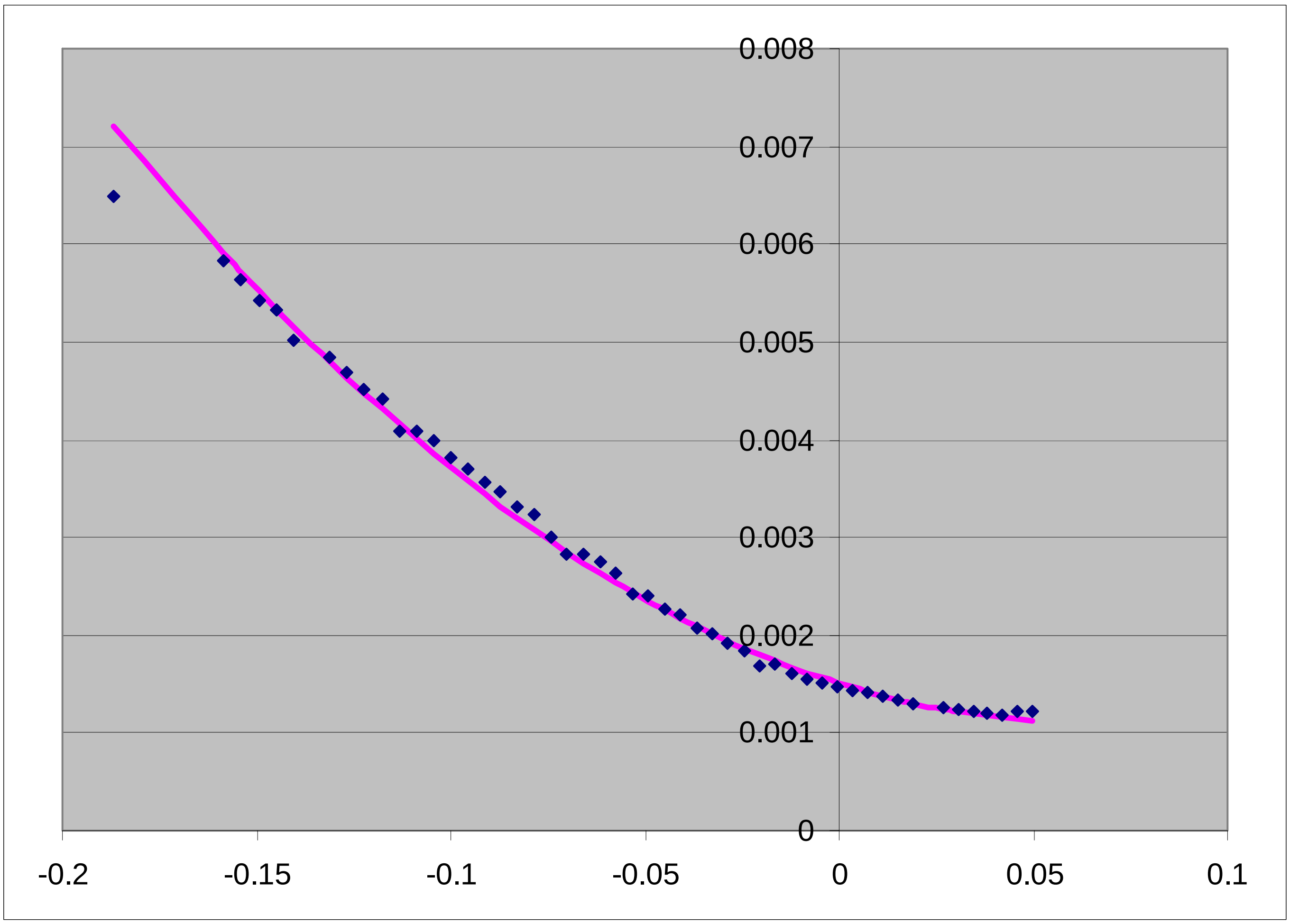
The resulting Delta curve is a set of points (δ, IV) where $\delta = \Delta$ for call and $\delta = 1 - |\Delta|$ for put options. It is convenient since now δ ranges within interval [0.1, 0.9] with step 0.05.

After all Delta curves are built, we separately handle expiries having flat curves (a=b=0), or, in other words, having only ATM volatility calculated. The data for them is interpolated/extrapolated using Delta curves with non-flat pattern first and then shifted to their initial ATM volatility level (interpolated curve is multiplied by a factor of initial ATM volatility / interpolated ATM volatility). We interpolate variance $\sigma^2 t$ linearly by time here; extrapolation by time is always flat.

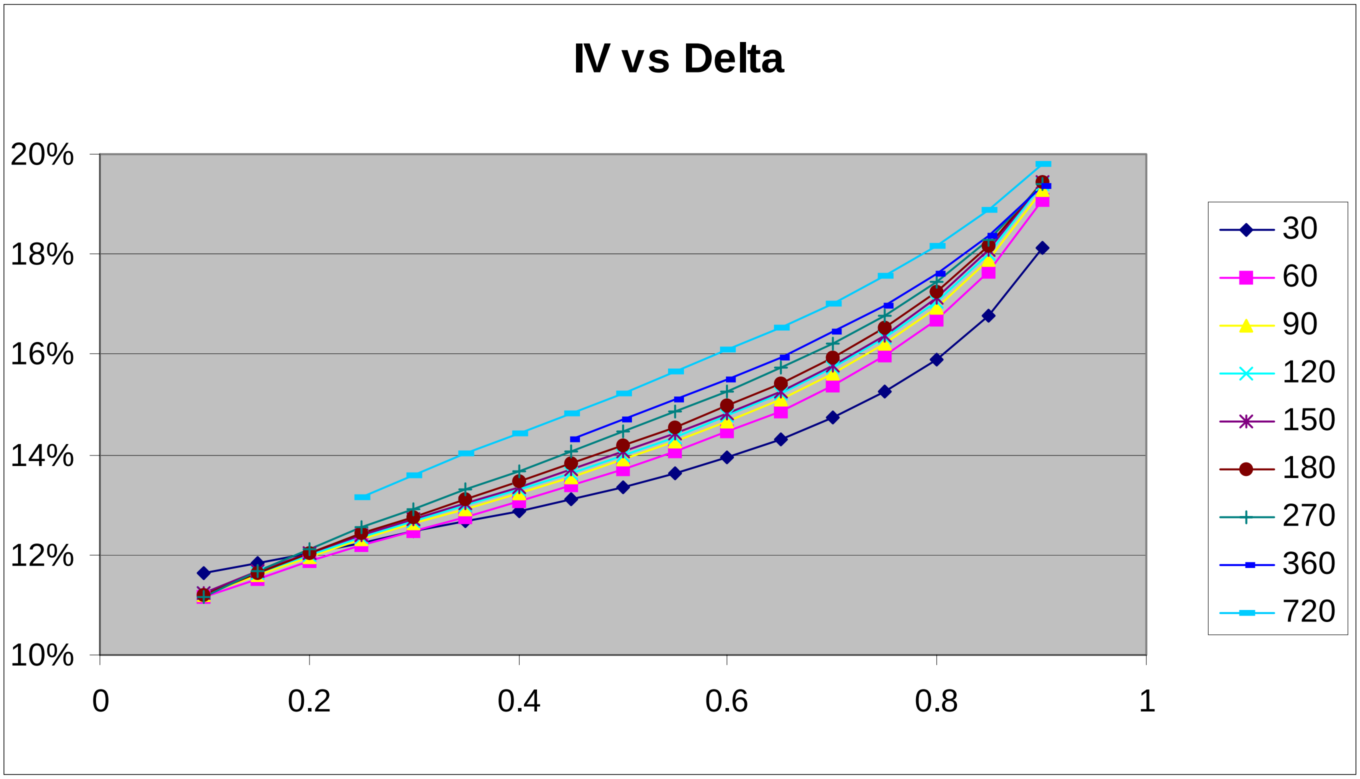
Finally, the Delta surface is built by interpolating IV from delta curves for all real market expirations to standard terms: 30, 60, 90, 120, 150, 180, 270, 360 and 720 days to expiry. For each term we interpolate variance $\sigma^2 t$ linearly in time between the nearest market expirations. Extrapolation by time is always flat.

Example

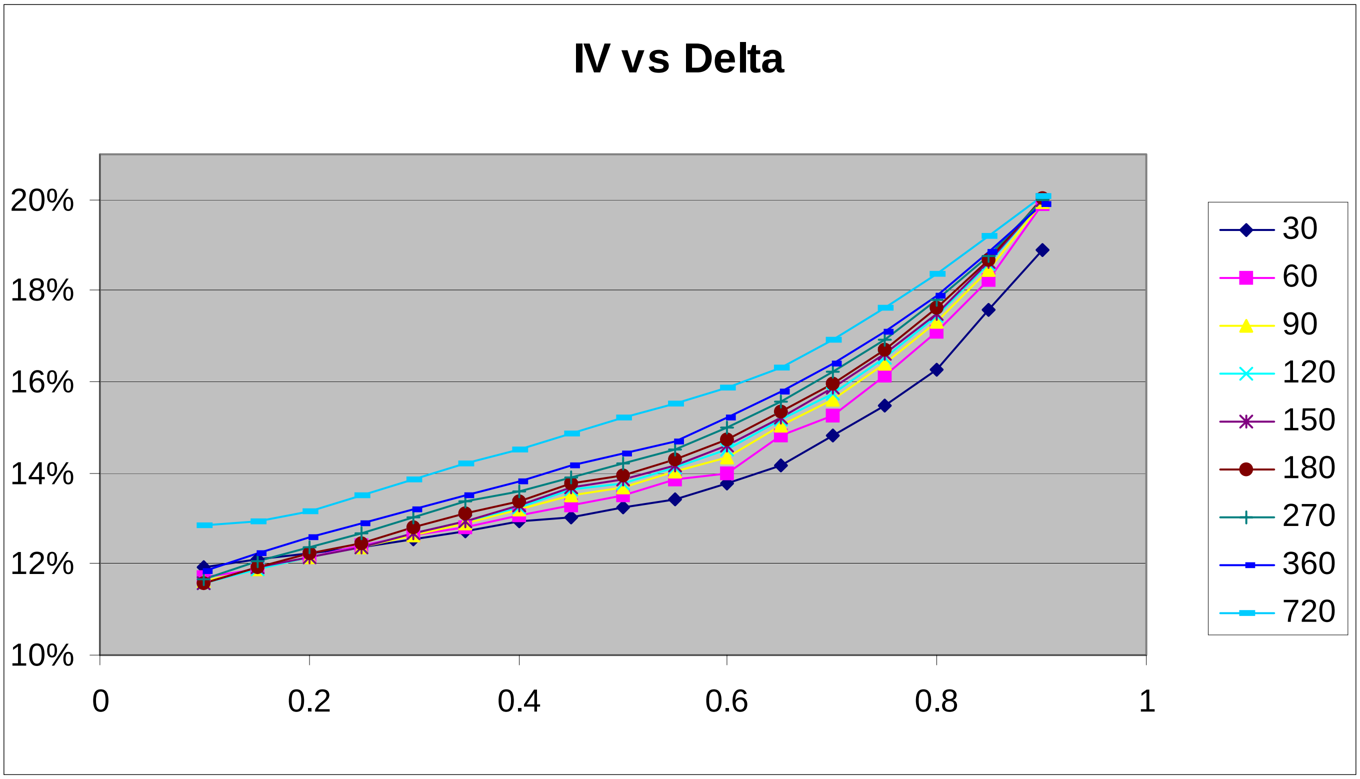
We illustrate our approach using Standard and Poor's 500 Index (SPX) data.
Consider July 19, 2006 SPX Index options with an expiration of Aug 2006 (31 days to expiry) and forward price at expiration of F=1265.62. Chart below shows market implied volatility data for out of the money options (dots) and Parameterized IV skew (line) $y = a x^2 + bx + c$, where x - log moneyness, y - implied variance:



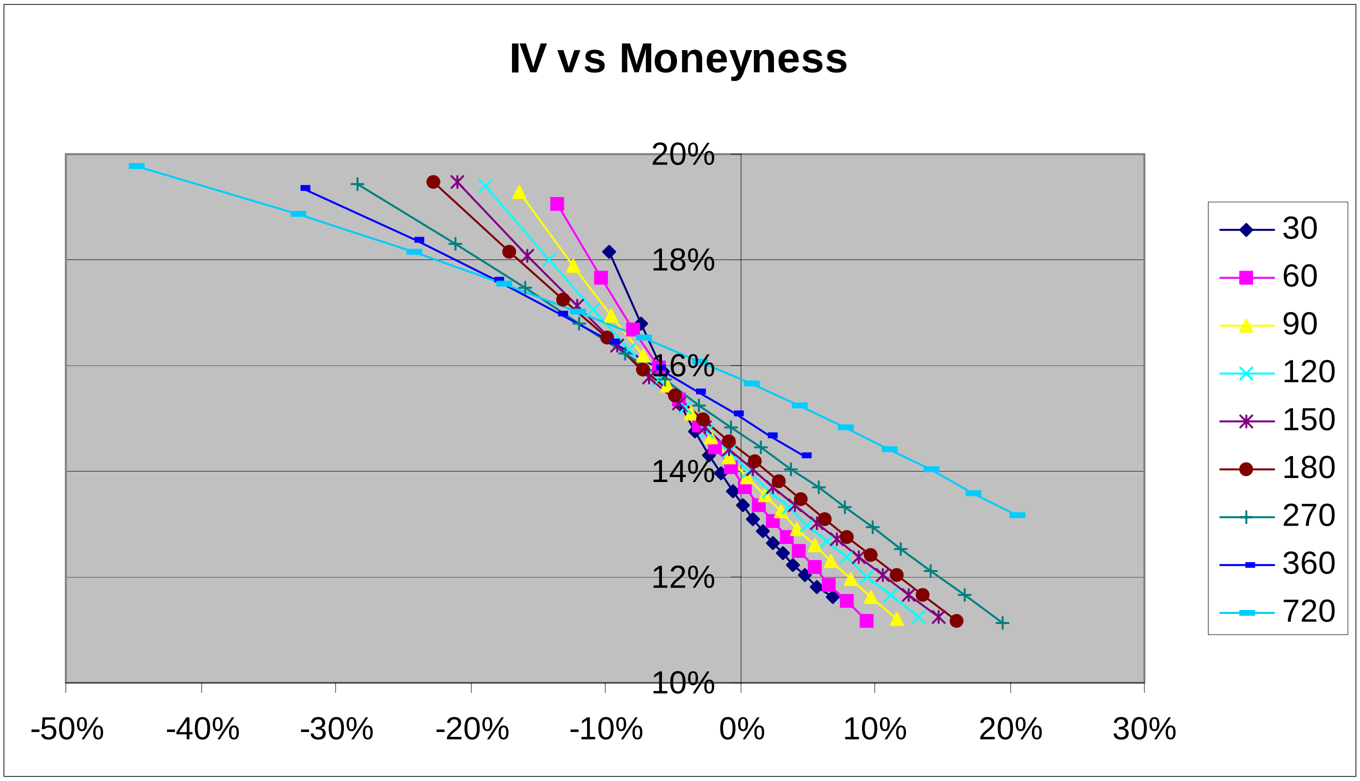
As you can see the Parameterized IV skew fits real market data well. Doing the same for other expiries and finding IV as a function of standard term and delta we'll get the Delta surface. Chart below shows IV as a function of delta for standard terms 30, 60, 90, 120, 150, 180, 270, 360 and 720 calendar days:



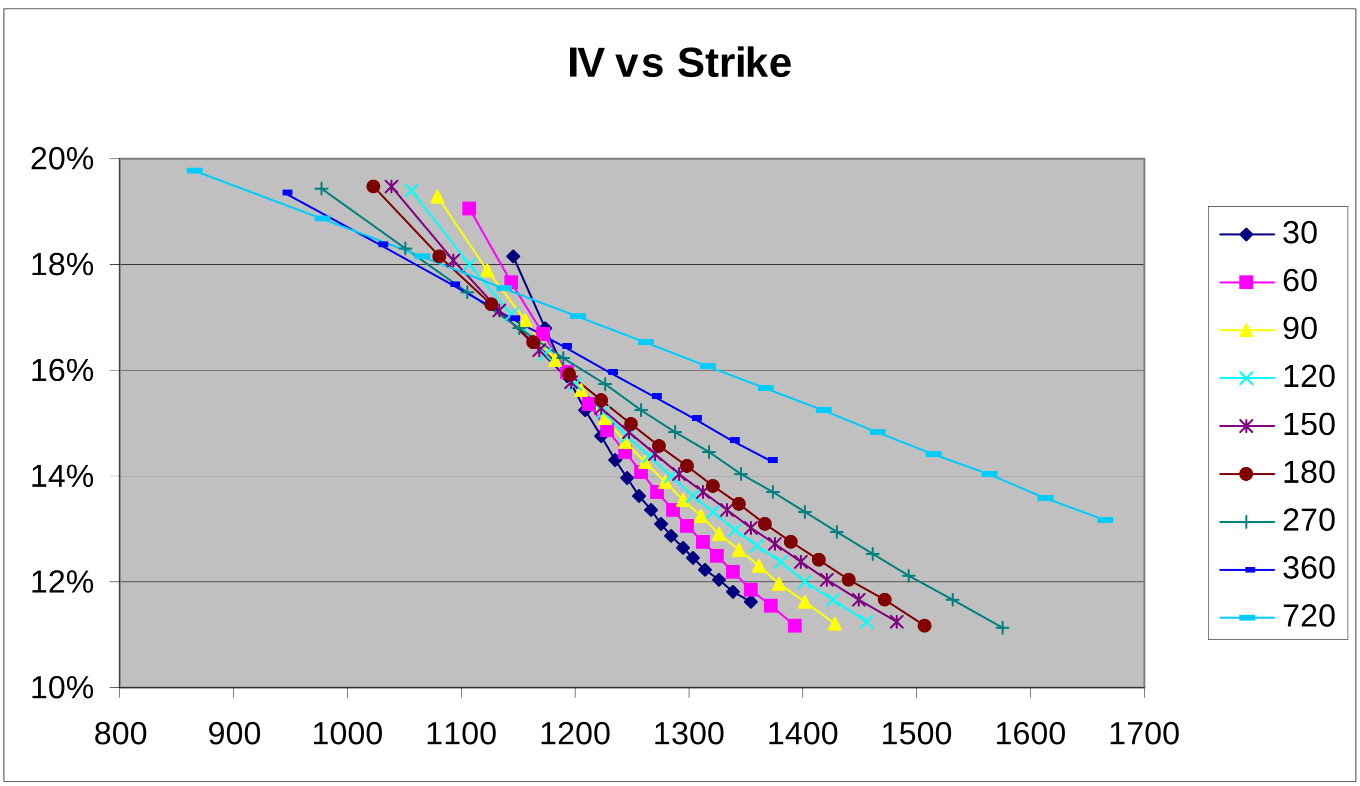
Comparing with Raw Delta surface for the same name and date (below), one will see that Delta surface built on parameterized data smooths occasional data irregularities ("noise"):



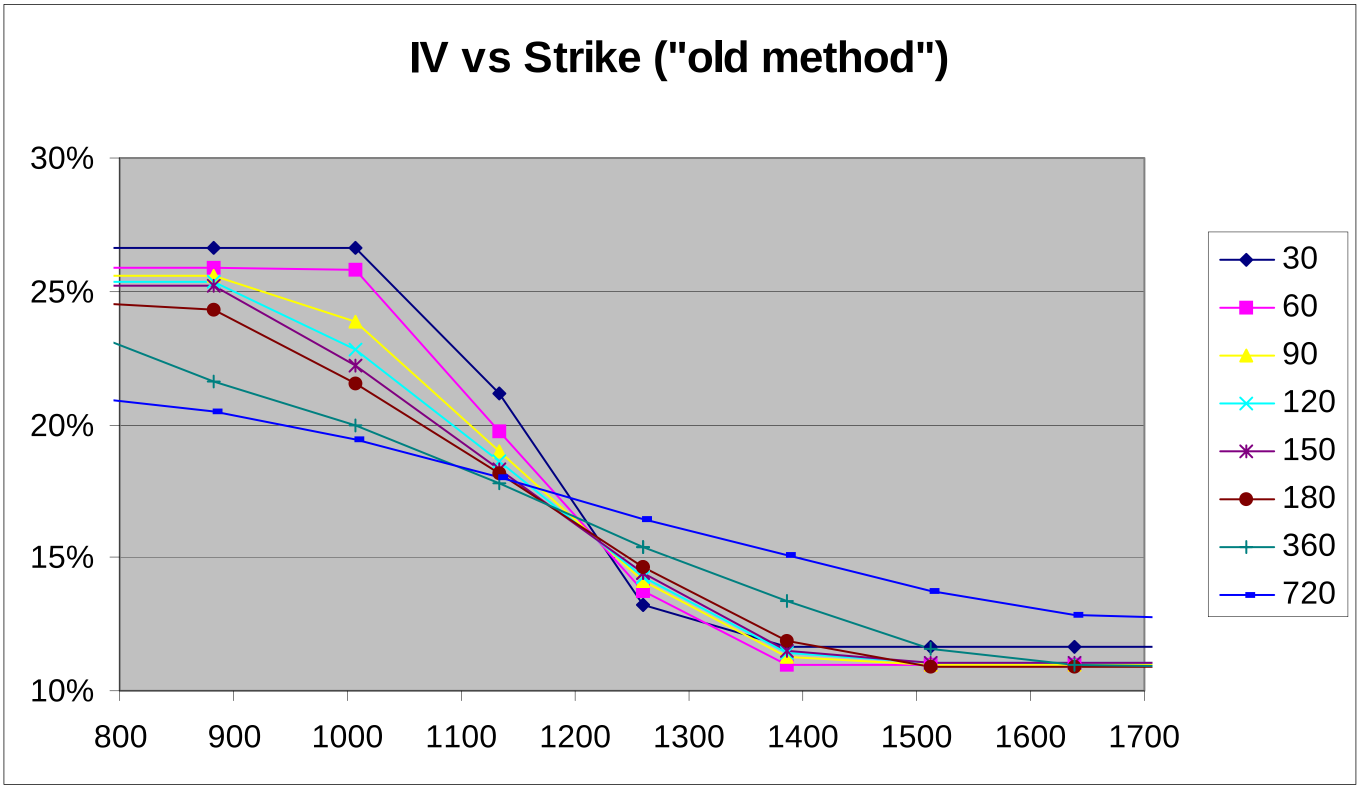
Using Delta surface or Raw Delta surface one can alternatively build a surface against log moneyness ...



... or against virtual strike:



It is interesting to compare this chart with the "standard" IV Surface by moneyness (the other dataset we've been calculating for years; Call/Put gap has been removed in the chart below):



It is seen that "old method" yields less accurate description for near-ATM strikes and somewhat too high IV values for low strikes.